Ecole Doctorale des Sciences Fondamentales

Title of the thesis:

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Summary : Linear dynamics is a very active branch of functional analysis. It studies the orbits of linear operators, namely the sets $\{T^n x ; n>0\}$ where T is a continuous linear operator acting on some Banach space X and x is a vector in X. For instance, one is interested in the existence of a dense orbit. This is intimely connected with the invariant subset problem, a famous open question in operator theory.

Linear dynamics involves many parts of analysis : operator theory of course, complex analysis for the study of examples and counterexamples, ergodic theory… Many questions around this subject remain open and in the thesis I plan to study two of them.

1. Universal operators for topological dynamics

One may think that linear dynamics is not as rich as the standard topological dynamics, since we work only with linear maps. Surprizingly, this is not the case : in a seminal paper, in 2000, N. Feldman has exhibited an example on an operator T acting on a Hilbert space H such that, given any dynamical system S :K->K, where K is compact, there exists a subset M of H, invariant by T, such that S is conjugated to the restriction of T to M.

Thus, this operator is called universal for topological dynamics. The first part of the thesis will be devoted to the study of universal operators in this sense. The first thing to be done is to find a nice sufficient condition for an operator to be universal. Hopefully, this would give new examples beyond Feldman's examples. It will also be interesting to study which properties such an operator should have.

2. Common hypercyclicity

We say that an operator T is hypercyclic as soon as it admits a vector x with dense orbit. The vector x is then called a hypercyclic vector for T. When we consider a family of hypercyclic operators (for instance, a family of translation operators or a family of weighted shifts), it is natural to ask whether it admits a common hypercyclic vector, namely a vector x which is hypercyclic for ll operators of the family.

This problem has been widely studied these last ten years. Very recently, important progresses have been made in two directions : when the operators belong to a semigroup, or when the operators are the multiples of the same operator. The

technics involved are rather different and the second part of the thesis will be dedicated to their study. The aim will be to combine them, in order to prove positive results for the existence of a common hypercyclic vector for all multiples of operators in a semigroup.